Algebraic Proof **Homework**

1. Disprove, by using a counter-example, that the sum of any two prime numbers is even.

2. Sandeep says that the sum of four consecutive numbers will always be a multiple of 4. Is she correct?

3. Disprove the following statement by providing a counter example:
   If $x > y$, then $x^2 > y^2$

4. Prove that $(x + y)^2 - (x - y)^2 = 4xy$

5. x is an integer. Prove that $(x + 2)^2 - (x - 2)^2$ is an even number.

6. Prove, algebraically, that the sum of two consecutive odd numbers is divisible by 4.

7. Prove, algebraically, that the sum of three consecutive integers is a multiple of 3.

8. Prove, algebraically, that the product of two consecutive integers will always be even.

9. Prove, algebraically, that the difference of any two consecutive square numbers is odd.

10. Prove, algebraically, that the expression $n^2 + 7n + 10$ will never be a prime number for all integer values $n > 0$.

Challenge

Prove, algebraically, that the sum of the squares of any three consecutive positive integers will always have a remainder of 2 when divided by 3.

WWW	**Algebraic Proof and Counter-Examples**			
1-3	Disproving a statement by providing a counter-example.			
4-5	Direct proof from a given algebraic statement.			
6-10	Direct proof from a worded statement.			
Challenge	More complicated direct proof example.			
EBI				

Answers

1. 2 + 3 = 5 (in fact, any statement which uses 2 as one of the primes will work).

2. No, since 1 + 2 + 3 + 4 = 10 which is not divisible by 4.

3. $x = 2$, $y = -3$ (y must be negative for this to work).

4. $x^2 + 2xy + y^2 - (x^2 - 2xy + y^2) = 4xy$

5. $x^2 + 4x + 4 - (x^2 - 4x + 4) = 8$
   $8x$ can be written as $2(4x)$ which means it is always even.

6. Let $2n + 1$ be an odd number, then $2n + 3$ is a consecutive odd number.
   $2n + 1 + 2n + 3 = 4n + 4 = 4(n + 1)$
   Therefore, divisible by 4.

7. Let n be an integer. Then $n + 1$ and $n + 2$ are two consecutive integers.
   $n + n + 1 + n + 2 = 3n + 3 = 3(n + 3)$
   Therefore, a multiple of 3.

8. Let n be an integer, then $n + 1$ is a consecutive number.
   $n(n + 1) = n^2 + n$
   If n is odd, then n^2 is odd, hence $n^2 + n$ is even.
   If n is even, then n^2 is even, hence $n^2 + n$ is even.

9. Let the first square number be n^2. Then the next square number is $(n + 1)^2$.
   $(n + 1)^2 - n^2 = n^2 + 2n + 1 - n^2$
   $= 2n + 1$
   $2n$ is always even, so $2n + 1$ is always odd.

10. $n^2 + 7n + 10 = (n + 5)(n + 2)$
    If $n > 0$ then $n + 5$ and $n + 2$ will always be positive integers.
    Hence $n^2 + 7n + 10$ can never be a prime number under these circumstances.

Challenge

Let the first square number be n^2. Then the next two square numbers are $(n + 1)^2$ and $(n + 2)^2$

$n^2 + (n + 1)^2 + (n + 2)^2 = n^2 + n^2 + 2n + 1 + n^2 + 4n + 4$

$= 3n^2 + 6n + 5$

$3n^2$ and $6n$ are both multiples of 3 and therefore will have no remainder when divided by 3. 5 has a remainder of 2 when divided by 3. Therefore $3n^2 + 6n + 5$ will always have a remainder of 2 when divided by 3.

Completing the Square **Homework**

1. Write $x^2 + 6x + 14$ in the form $(x + a)^2 + b$, where a and b are integers.
2. Write $x^2 – 4x – 2$ in the form $(x + a)^2 + b$, where a and b are integers.
3. Write $x^2 + 3x + 4$ in the form $(x + a)^2 + b$, where a and b are constants.
4. Write $x^2 - 7x – 9$ in the form $(x + a)^2 + b$, where a and b are constants.
5. Write $3x^2 + 12x + 21$ in the form $p(x + q)2 + r$, where p, q and r are constants.
6. Write $2x^2 + 10x + 14$ in the form $p(x + q)^2 + r$, where p, q and r are constants.
7. Write $3x^2 + 9x + 8$ in the form $p(x + q)^2 + r$, where p, q and r are constants.
8. Robbie completes the square for the expression $x^2 + bx + c$ and achieves the expression $(x + 5)^2 – 12$. Find the values of b and c.
9. Complete the square for the equation $y = x^2 + 4x + 11$ to find the coordinates of the turning point and state whether it is a minimum or maximum.
10. Complete the square for the equation $y = -x^2 – 6x + 4$ to find the coordinates of the turning point and state whether it is a minimum or maximum.
11. Complete the square for the equation $y = 3x^2 + 9x – 1$ to find the coordinates of the turning point and state whether it is a minimum or maximum.

Challenge

Use completing the square to solve the equation $x^2 + 4x – 7 = 0$, giving your answers in surd form.

WWW	**Completing the Square**			
1-4, 8	Completing the square when the coefficient of x^2 is 1.			
5-7	Completing the square when the coefficient of x^2 is not 1.			
9-11	Finding the turning points.			
Challenge	Solving use completed square form.			
EBI				

Answers

1. $(x + 3)^2 + 5$
2. $(x - 2)^2 - 6$
3. $(x + 1.5)^2 + 1.75$
4. $(x - 3.5)^2 - 21.25$
5. $3(x + 2)^2 + 9$
6. $2(x + 2.5)^2 + 1.5$
7. $3(x + 1.5)^2 + 1.25$
8. $b = 10$, $c = 13$
9. $(x + 2)^2 + 7$
   (-2, 7), minimum
10. $-(x + 3)^2 + 13$
    (-3, 13), maximum
11. $3(x + 1.5)^2 - 7.75$
    (-1.5, -7.75), minimum

Challenge

$x = -2 + \sqrt{11}$

$x = -2 - \sqrt{11}$

Functions **Homework**

1. A function f is defined as $f(x) = 3x + 7$. Find the value of:
    a. $f(4)$
    b. $f(-6)$
    c. $f(-1)$
    d. $f(8)$
2. A function g is defined as $g(x) = 2x^3$. Find the value of:
    a. $g(2)$
    b. $g(-3)$
    c. $g(4)$
    d. $g(0.5)$
3. A function f is defined as $f(x) = 2x - 1$. Find the value of a such that $f(a) = 9$.
4. A function g is defined as $g(x) = 4x + 2$. Find $g^{-1}(x)$, giving your answer in terms of x.
5. A function f is defined as $f(x) = \frac{x + 3}{2}$. Find $f^{-1}(x)$, giving your answer in terms of x.
6. A function f is defined as $f(x) = x^2$.
   A function g is defined as $g(x) = 2x$
   Find:
    a. $fg(x)$
    b. $gf(x)$
    c. $gg(x)$
    d. $fg(6)$

Challenge

Functions f, g and h are defined as:

$f(x) = 4x$

$g(x) = x - 4$

and $h(x) = 2x + 3$

Find $fgh(x)$, giving your answer in terms of x.

WWW	**Functions**			
1, 2, 3	Evaluating functions.			
4, 5	Inverse functions.			
6, 7	Composite functions.			
Challenge	Composite functions.			
EBI				

Answers

1.
   a. 19
   b. -11
   c. 4
   d. 31
2.
   a. 16
   b. -54
   c. 128
   d. $\frac{1}{4}$
3. $a = 5$
4. $g^{-1}(x) = \frac{x - 2}{4}$
5. $f^{-1}(x) = 2x - 3$
6.
   a. $4x^2$
   b. $2x^2$
   c. $4x$
   d. 144

Challenge

$fgh(x) = 8x - 4$

Functions **Homework**

1. A function f is defined as $f(x) = 3x - 4$. Find the value of:

   a. $f(4)$

   b. $f(-6)$

   c. $f(-1)$

   d. $f(0.5)$

2. A function g is defined as $g(x) = 2x^3$. Find the value of:

   a. $g(2)$

   b. $g(-3)$

   c. $g(5)$

   d. $g(-\frac{1}{4})$

3. A function f is defined as $f(x) = \frac{2x - 1}{3}$. Find the value of a such that $f(a) = a$.

4. A function g is defined as $g(x) = \frac{x + 3}{2x}$. Find $g^{-1}(x)$, giving your answer in terms of x.

5. A function f is defined as $f(x) = \frac{x - 1}{x + 2}$. Find $f^{-1}(x)$, giving your answer in terms of x.

6. A function f is defined as $f(x) = 4x$

   A function g is defined as $g(x) = 2x + 5$

   Find the value of x such that $fg(x) = 36$

7. A function f is defined as $f(x) = \frac{2x - 1}{3}$

   A function g is defined as $g(x) = \frac{x + 3}{2x}$

   Find $gf(x)$, giving your answer in terms of x and in its simplest form.

Challenge

Functions f, g and h are defined as:

$f(x) = 4x$

$g(x) = x - 4$

and $h(x) = 2x + 3$

Find $fgh(x)$, giving your answer in terms of x.

WWW	**Functions**			
1, 2, 3	Evaluating functions.			
4, 5	Inverse functions.			
6, 7	Composite functions.			
Challenge	Composite functions.			
EBI				

Answers

1.
   a. 8
   b. -22
   c. -7
   d. -2.5
2.
   a. 16
   b. -54
   c. 250
   d. $-\frac{1}{32}$
3. $a = -1$
4. $g^{-1}(x) =$
5. $f^{1}(x) =$ $\frac{3}{2x-1}$
6. $x = 2$ $\frac{2x+1}{1-x}$
7. $gf(x) =$

$\frac{x+4}{2x-1}$

Challenge

$fgh(x) = 8x - 4$

Transforming Functions **Homework**

1. For a function $y = f(x)$, describe the transformations given by:

    2. $y = f(x) + 2$

    3. $y = f(x - 3)$

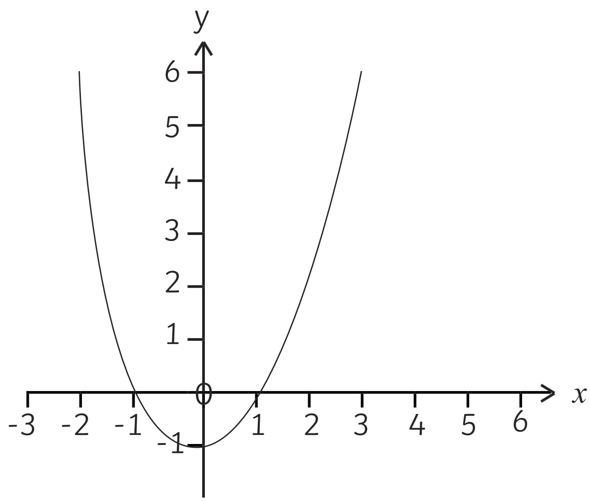
    4. $y = f(x + 1) + 4$

    5. $y = -f(x - 4)$

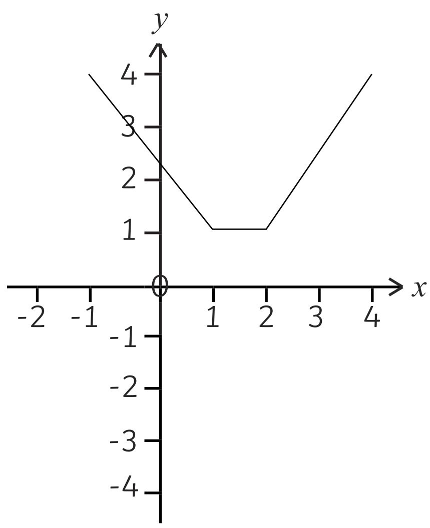
    6. $y = f(-x) + 3$

    7. $y = f(-x + 2) - 1$

8. The graph of $y = f(x)$ is shown. Sketch, on the same axes, the graph of $y = f(x - 2) + 4$.



9. The graph of $y = f(x)$ is shown. Sketch, on the same axes, the graph of $y = -f(x) + 1$.

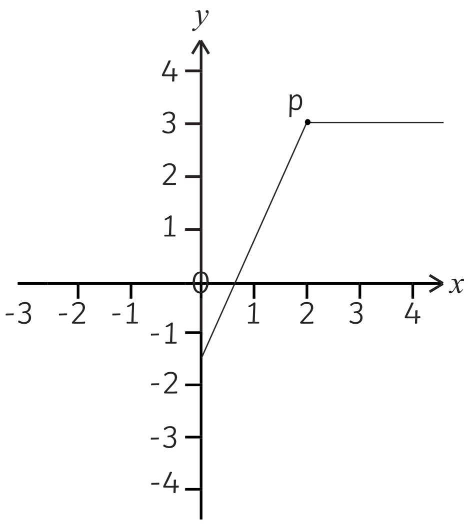


10. The graph of y = $f(x)$ is shown. The point P has coordinates (2, 3). Find the coordinates of the point P when the graph is transformed to:

11. $y = f(x + 1) - 2$

12. $y = f(-x + 5)$

13. $y = -f(x - 1) + 4$



Challenge

For the graph $y = \sin(x)$, write down the equation for the graph given by a transformation along the vector $\begin{pmatrix}90\\-1\end{pmatrix}$

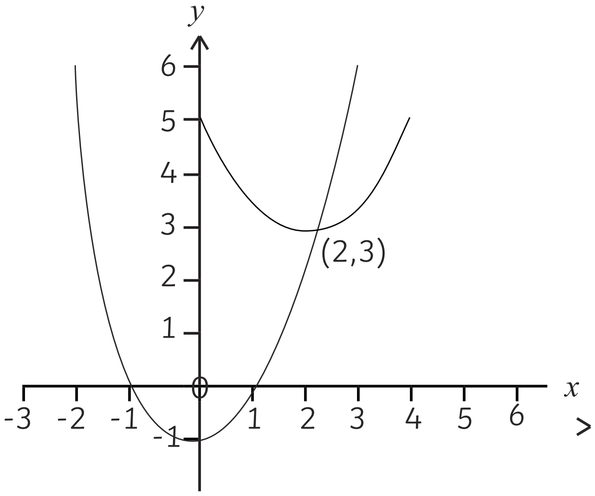
WWW	Transforming Functions			
1	Identify transformations.			
2, 3	Sketching transformations.			
4	Interpreting transformed coordinates.			
Challenge	Transforming trigonometric functions.			
EBI				

Answers

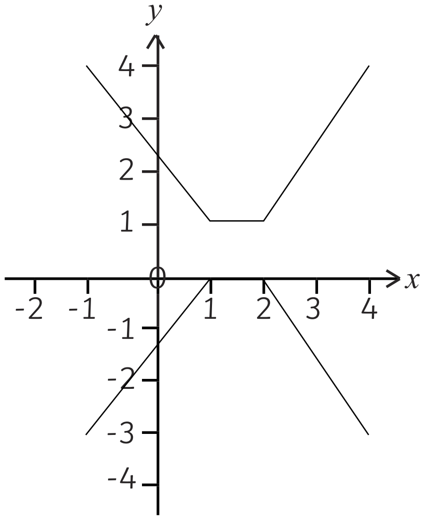
1.

2. translation by the vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
3. translation by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$
4. translation by the vector $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$
5. translation by the vector $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ and reflected in the x-axis.
6. reflection in the y-axis and translated by the vector $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$
7. reflection in the y-axis, then translated by the vector $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$

8.



9.



10.

11. (1, 1)

12. (-7, 3)

13. (3, 1)

Challenge

$y = \sin(x - 90) - 1$

Transforming Functions **Homework**

1. For a function $y = f(x)$, describe the transformations given by:

    2. $y = f(x) + 2$

    3. $y = f(x) - 3$

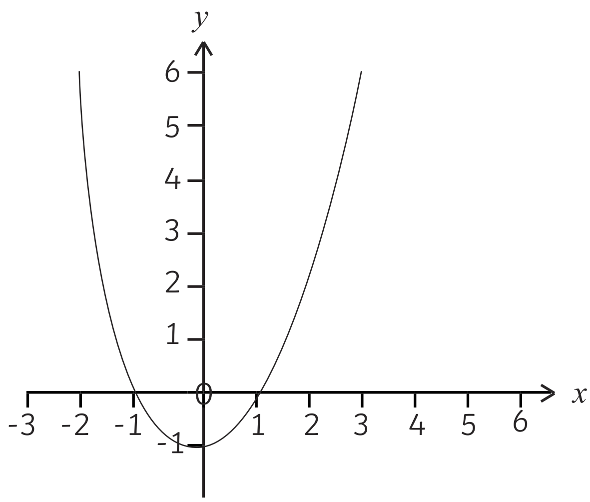
    4. $y = f(x + 1)$

    5. $y = f(x - 4)$

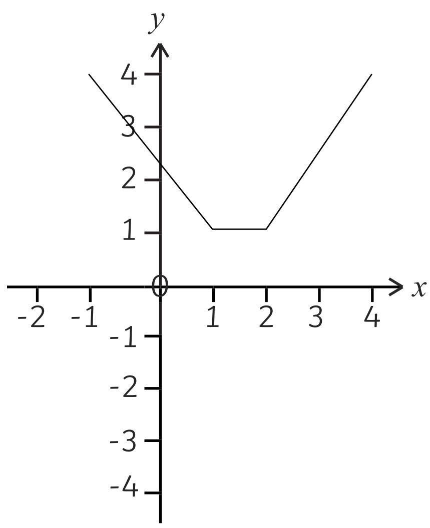
    6. $y = f(-x)$

    7. $y = f(x + 3) - 1$

8. The graph of $y = f(x)$ is shown. Sketch, on the same axes, the graph of $y = f(x) + 4$.



9. The graph of $y = f(x)$ is shown. Sketch, on the same axes, the graph of $y = -f(x)$.

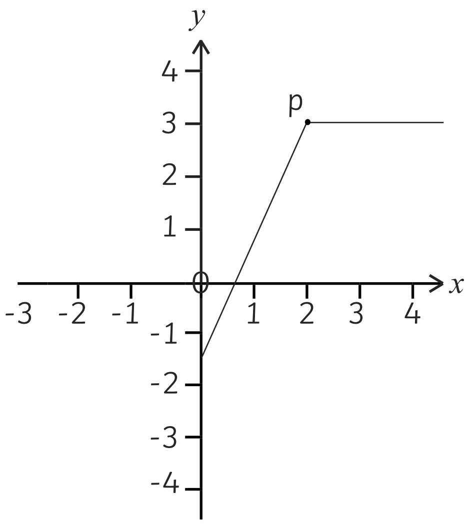


10. The graph of $y = f(x)$ is shown. The point P has coordinates (2, 3). Find the coordinates of the point P when the graph is transformed to:

11. $y = f(x) - 2$

12. $y = f(x + 5)$

13. $y = f(x - 1) + 4$



Challenge

For the graph $y = f(x)$ shown in question 4, write down the coordinates of the point P when the graph is transformed to $y = -f(x + 3) - 7$

WWW	**Transforming Functions**	🙂	😐	🙁
1	Identify transformations.			
2, 3	Sketching transformations.			
4	Interpreting transformed coordinates.			
Challenge	Trickier transformed coordinates.			
EBI				

Answers

1.

2. translation by the vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

3. translation by the vector $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

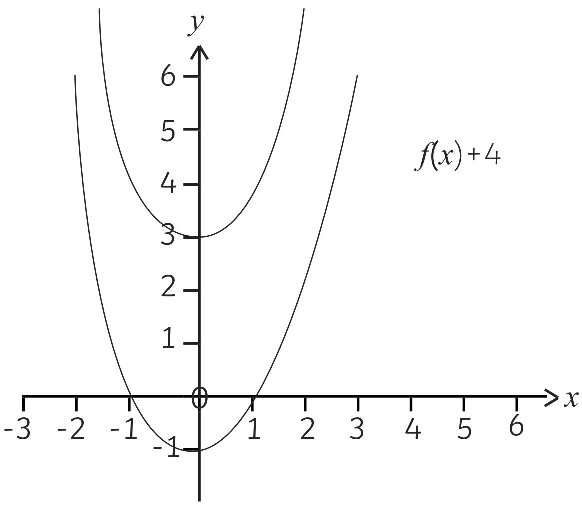
4. translation by the vector $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

5. translation by the vector $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

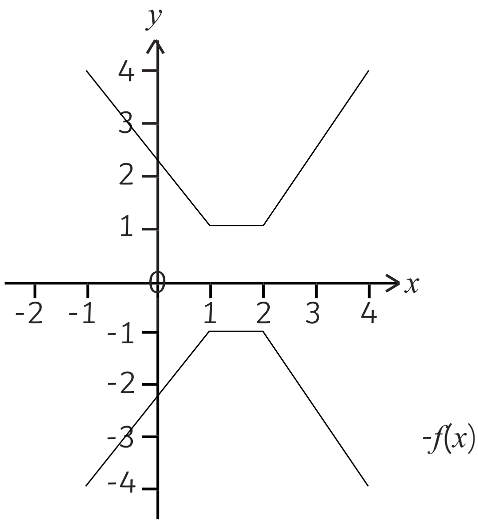
6. reflection in the y-axis

7. translation by the vector $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$

8.



9.



10.

   11. (2, 1)
   12. (-3, 3)
   13. (3, 7)

Challenge

(-1,-10)